

$1/N_c$ Corrections in Meson-Baryon scattering

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ABSTRACT: Corrections to meson/ground-state baryon scattering amplitudes in the $1/N_c$ expansion of QCD have previously been shown to be controlled by the t -channel difference $|I_t - J_t|$ of isospin and angular momentum and by the change of hypercharge Y_t . Here we derive the corresponding expressions in the original scattering s channel, allowing for nonzero meson spin and nontrivial SU(3) flavor quantum numbers, and provide explicit examples of the crossing relevant for $\pi N \rightarrow \rho N$ and KN scattering.

KEYWORDS: $1/N$ Expansion, QCD.

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1. Introduction

The $1/N_c$ expansion of large N_c QCD, where N_c is the number of color charges, remains one of the very few predictive model-independent schemes for studying strong interaction physics. It is particularly useful for studies of baryons, which require a content of N_c valence quarks in order to form color singlets. Irreducible operators containing many quark lines tend to be suppressed through powers of $\alpha_s = O(1/N_c)$, meaning therefore that the $1/N_c$ expansion produces an effective field theory for baryonic systems. This simple fact has been exploited to great effect for many years. The original examples using this “operator” technique [1] considered static baryon properties for baryons stable under strong decays (or, more accurately, baryons whose widths vanish as a power of $1/N_c$). However, there is no reason to expect baryonic dynamical properties, such as those probed in meson-baryon scattering, to follow immediately from a static operator approach.

Substantial progress derives from employing group theoretical structures inspired by the Skyrme and other chiral soliton models, since these models provide a natural way to couple baryons to chiral mesons and to represent scattering processes. Inspired by the famous Adkins-Nappi-Witten papers [2], the Siegen group [3] and Mattis and collaborators [4, 5] exploited this group theory to great phenomenological and formal effect in the 1980s. The key quantity in these analyses is a conserved “grand spin” $\mathbf{K} = \mathbf{I} + \mathbf{J}$, which in soliton models characterizes hedgehog states. However, since I and J but not K are

externally measured quantum numbers, the physical baryon state consists of a linear combination of K eigenstates; in this approach K is therefore a good but hidden quantum number.

Nevertheless, the direct connection of these calculations to the large N_c limit, based partly upon the observation that baryons at large N_c are heavy and therefore semiclassical objects with the right properties to be represented as solitons, remained indirect. One indication of this connection arises from the completely spin-flavor symmetric quantum numbers of the lowest-lying baryons such as N and Δ (the “ground-state band”), which is what one would expect from states constructed entirely from a number (N_c) of $K=0$ hedgeholic quarks.

Full compatibility with the large N_c limit at the hadronic level was shown in ref. [6] to arise from ingredients already present in the literature but not previously assembled. First, in the mid-1980s Donohue noted [7] a great simplification of the structure of meson-baryon scattering amplitudes when considered in the t rather than s channel: The leading-order in $1/N_c$ amplitudes for meson-baryon scattering [which are $O(N_c^0)$] have equal t -channel isospin and angular momentum exchange quantum numbers, $I_t = J_t$. Mattis and Mukerjee then showed [5] that a group-theoretical crossing of the $I_t = J_t$ rule from the t into the s channel is equivalent to the ansatz of underlying K conservation. Years later, and in a different context, Kaplan and Savage, and Kaplan and Manohar (KSM) [8] showed that the Dashen-Jenkins-Manohar consistency conditions [9], which amount to imposing unitarity order by order in powers of $1/N_c$ in meson-baryon scattering, lead to the $I_t = J_t$ rule. Finally, just a few years ago, ref. [6] assembled these facts to show that the meson-baryon scattering results based upon the group theory of chiral soliton models are in fact true results of the large N_c limit. Subsequent work showed [10] that not only does the $I_t = J_t$ rule also apply to 3-flavor processes, but also $Y_t = 0$ at leading N_c order, due to interesting properties of SU(3) Clebsch-Gordan coefficients (CGC) as N_c grows large.

A recent and extensive body of literature [6, 10–18] builds upon not only this connection to large N_c , but also upon the observation that having an underlying conserved quantum number contributing to multiple partial waves means that a resonant pole of a given mass and width appearing in one partial wave of given I, J^P quantum numbers also appears in numerous others, giving rise to multiplets of baryon resonances degenerate in mass and width. Here, however, we are more interested in the structure of the amplitudes themselves in terms of the $1/N_c$ expansion, rather than poles that lie within them.

In particular, ref. [11] pointed out that the KSM approach also implies that amplitudes with $|J_t - I_t| = n$ are suppressed by at least $1/N_c^n$ compared to leading order, which provides a way to study $1/N_c$ -suppressed amplitudes — albeit expressed in terms of t -channel quantities. This classification was employed phenomenologically to study $\pi N \rightarrow \pi N, \pi \Delta$ [11], pion photoproduction [12], and chiral threshold effects [13], but always for spinless, non-strange mesons. Generalizing this approach to mesons with spin (such as ρ) and mesons with strangeness (such as K) are among the goals of this work.

Moreover, in all previous cases the subleading amplitudes were expressed in terms of t -channel quantum numbers, even though the s channel is the most natural from the point of view of representing the data (e.g., baryon resonances are identified in this channel). It is

clearly highly desirable to express all amplitudes, including these subleading effects, directly in terms of s -channel quantum numbers. Deriving the exact nature of this transformation constitutes the central goal of this paper.

The path to this goal may seem highly technical and mathematical, but it has a simple physical significance. Meson-baryon scattering can be described in terms of a $1/N_c$ expansion, but previously the only convenient way for distinguishing the various orders of $1/N_c$ corrections to the leading-order [$O(N_c^0)$] result was by describing the process in terms of t -channel quantities. Here we carry out the crossing of the amplitudes into s -channel quantities in such a manner that identifies quantum numbers whose changes correlate with specific orders in the $1/N_c$ expansion, and as a result never lose sight of where the various t -channel quantities contribute.

This paper is organized as follows: In section 2 we define the relevant quantities of the scattering process. Section 3 presents the original scattering amplitude expressions in the large N_c limit in terms of both s -channel and t -channel quantum numbers. In section 4 we obtain the relations for crossing between s -channel and t -channel scattering descriptions, independent of the $1/N_c$ expansion. Section 5 explains how to incorporate $1/N_c$ corrections to meson-baryon scattering processes in terms of t -channel quantum numbers. Section 6 merges these ideas and shows how $1/N_c$ corrections may be expressed in the original s -channel language, providing special cases and examples. Finally, section 7 summarizes.

2. Observables

We consider meson-baryon scattering processes denoted by

$$\phi + B \rightarrow \phi' + B'. \tag{2.1}$$

Here, ϕ (ϕ') is a meson of spin S_ϕ ($S_{\phi'}$) in the state of the flavor SU(3) representation R_ϕ ($R_{\phi'}$) with isospin I_ϕ ($I_{\phi'}$) and hypercharge Y_ϕ ($Y_{\phi'}$). B (B') is a baryon of spin S_B ($S_{B'}$) in the state of the flavor SU(3) representation R_B ($R_{B'}$) with isospin I_B ($I_{B'}$) and hypercharge Y_B ($Y_{B'}$) within the ground-state spin-flavor multiplet [the completely spin-flavor symmetric large N_c generalization of the SU(6) **56**, for which the nonstrange members N , Δ , etc., have $I_B = S_B$ and $Y_B = \frac{N_c}{3}$]. The hadrons possess relative orbital angular momentum L (L'), and their total spin angular momentum (not including relative orbital angular momentum) is denoted S (S'). Let us additionally label the meson total angular momentum J_ϕ ($J_{\phi'}$), and define the grand spin K as the vector sum of I_ϕ and J_ϕ , and similarly for K' . Auxiliary quantum numbers \tilde{K} (\tilde{K}') label the vector sums of L and I_ϕ (L' and $I_{\phi'}$).

We also label the intermediate compound s -channel state by total quantum numbers J_s , R_s , I_s , and Y_s . The representation R_s formed from $R_B \otimes R_\phi$ sometimes occurs more than once in the product (for example, $\mathbf{8} \otimes \mathbf{8}$ contains two $\mathbf{8}$'s), and this degeneracy quantum number — which need not be the same in the initial and final state — is denoted by γ_s (γ'_s). Lastly, we define the compound t -channel quantum numbers I_t and J_t . Classically, I_t and J_t are vector differences of isospins and spins, respectively, of the incoming and outgoing baryons; however, the simple difference $\mathbf{J}_1 - \mathbf{J}_2$ of two SU(2) vector operators does not also

transform as a vector operator. Nevertheless, the proper generalization is well known [19]: The notation $-\mathbf{J}$ is used as shorthand for the time-reversed form $\tilde{\mathbf{J}}$, an operator whose eigenstates are related to those ($|JJ_z\rangle$) of \mathbf{J} by $(-1)^{J+J_z} |J, -J_z\rangle$. Using this notation, the complete set of definitions reads

$$\begin{aligned}
 \mathbf{I}_s &\equiv \mathbf{I}_B + \mathbf{I}_\phi = \mathbf{I}_{B'} + \mathbf{I}_{\phi'} , \\
 \mathbf{S} &\equiv \mathbf{S}_B + \mathbf{S}_\phi , \quad \mathbf{S}' \equiv \mathbf{S}_{B'} + \mathbf{S}_{\phi'} , \\
 \mathbf{J}_\phi &\equiv \mathbf{L} + \mathbf{S}_\phi , \quad \mathbf{J}_{\phi'} \equiv \mathbf{L}' + \mathbf{S}_{\phi'} , \\
 \mathbf{J}_s &\equiv \mathbf{L} + \mathbf{S} = \mathbf{J}_\phi + \mathbf{S}_B = \mathbf{L}' + \mathbf{S}' = \mathbf{J}_{\phi'} + \mathbf{S}_{B'} , \\
 \tilde{\mathbf{K}} &\equiv \mathbf{L} + \mathbf{I}_\phi , \quad \tilde{\mathbf{K}}' \equiv \mathbf{L}' + \mathbf{I}_{\phi'} , \\
 \mathbf{K} &\equiv \tilde{\mathbf{K}} + \mathbf{S}_\phi = \mathbf{I}_\phi + \mathbf{J}_\phi = \tilde{\mathbf{K}}' + \mathbf{S}_{\phi'} = \mathbf{I}_{\phi'} + \mathbf{J}_{\phi'} , \\
 \mathbf{I}_t &\equiv -\mathbf{I}_B + \mathbf{I}_{B'} = \mathbf{I}_\phi - \mathbf{I}_{\phi'} , \\
 \mathbf{J}_t &\equiv -\mathbf{S}_B + \mathbf{S}_{B'} = \mathbf{J}_\phi - \mathbf{J}_{\phi'} ,
 \end{aligned} \tag{2.2}$$

while hypercharges are additive,

$$\begin{aligned}
 Y_s &\equiv Y_B + Y_\phi = Y_{B'} + Y_{\phi'} \\
 Y_t &\equiv -Y_B + Y_{B'} = Y_\phi - Y_{\phi'} ,
 \end{aligned} \tag{2.3}$$

and $(R_s, \gamma_s) \in R_B \otimes R_\phi$, $(R_s, \gamma'_s) \in R_{B'} \otimes R_{\phi'}$. Strictly speaking, the equality of initial- and final-state operators (such as for \mathbf{I}_s) indicates the presence of conservation laws; one may define, for example, a distinct \mathbf{I}'_s operator, but barring explicit isospin violation in the scattering process, any calculation shows the two operators to have the same effect. Moreover, all of the definitions sum two SU(2) vector operators that commute, a condition necessary in order for the resulting sum to obey canonical commutation relations among its components. The order of summands for each definition has been carefully chosen to reflect the order in which states are to be coupled: The couplings of $|J_1 J_{1z}\rangle |J_2 J_{2z}\rangle$ and $|J_2 J_{2z}\rangle |J_1 J_{1z}\rangle$ into a state $|JJ_z\rangle$ differ by the phase $(-1)^{J_1+J_2-J}$, while more nontrivial phases arise in SU(3) [20].

The definitions of eq. (2.2) must be considered carefully from a physical point of view, because going beyond the large N_c limit introduces recoil effects for the baryons. Only when the baryons are considered very heavy, in which case the center-of-mass and rest frames of B and B' coincide, does $\mathbf{J}_t \equiv -\mathbf{S}_B + \mathbf{S}_{B'}$ indicate the full change of angular momentum of the baryon in the t channel. A full relativistic treatment, as would be relevant to the most general case, employs a helicity formalism; nevertheless, the quantum numbers defined in eq. (2.2) (particularly J_t) continue to be well defined, even if their physical interpretation is not so simple as in the heavy-baryon limit. Indeed, this is one motivation for re-expressing the t -channel amplitudes in terms of the more familiar s -channel quantum numbers.

3. Scattering amplitude relations

The original derivations [3–5] of linear relations among meson-baryon scattering amplitudes rely upon solitonic representations of the baryon wave function. In particular, the

underlying state is given by a hedgehog configuration, whose functional dependence upon coordinates appears only through the characteristic mixed space-isospin inner product $\hat{\mathbf{r}} \cdot \boldsymbol{\tau}$; it is therefore an eigenstate of neither spin nor isospin separately, but rather the vector sum $\mathbf{K} \equiv \mathbf{I} + \mathbf{J}$ of the two. Each value of K gives rise to a distinct soliton configuration, which may be probed through scattering with mesons; a distinct *reduced* amplitude τ occurs for each K and initial (final) value L (L') of relative orbital angular momentum. If the meson probes also carry spin, one must also include the auxiliary quantum numbers \tilde{K} , \tilde{K}' defined in eq. (2.2), and if they carry strangeness, then one must also include isospin and hypercharge quantum numbers. The most general such reduced amplitude carries the labels $\tau_{K\tilde{K}\tilde{K}'LL'}^{\{II'Y\}}$.

From here it is a straightforward albeit tedious exercise to use the definitions in eq. (2.2) for coupling all appropriate quantum numbers to represent a full physical S matrix scattering amplitude $S_{LL'SS'J_s R_s \gamma_s \gamma'_s I_s Y_s}$ (dependence upon particular B, B', ϕ, ϕ' quantum numbers being implicit). The physical baryon state is given by the linear combination of solitonic configurations such that the composite state is the appropriate eigenstate of isospin and spin. $S_{LL'SS'J_s R_s \gamma_s \gamma'_s I_s Y_s}$ is reduced (i.e., independent of I_z, J_z quantum numbers) in the sense of the Wigner-Eckart theorem. The full expression, first derived in ref. [5] and corrected in ref. [14], reads

$$\begin{aligned}
 S_{LL'SS'J_s R_s \gamma_s \gamma'_s I_s Y_s} &= (-1)^{S_B - S_{B'}} ([R_B][R_{B'}][S][S'])^{1/2} / [R_s] \sum_{\substack{I \in R_\phi, I' \in R_{\phi'}, \\ I'' \in R_s, Y \in R_\phi \cap R_{\phi'}}} (-1)^{I+I'+Y} [I''] \\
 &\times \left(\begin{array}{c} R_B \quad R_\phi \parallel R_s \gamma_s \\ S_B \frac{N_c}{3} \quad IY \parallel I'' Y + \frac{N_c}{3} \end{array} \right) \left(\begin{array}{c} R_B \quad R_\phi \parallel R_s \gamma_s \\ I_B Y_B \quad I_\phi Y_\phi \parallel I_s Y_s \end{array} \right) \\
 &\times \left(\begin{array}{c} R_{B'} \quad R_{\phi'} \parallel R_s \gamma'_s \\ S_{B'} \frac{N_c}{3} \quad I'Y \parallel I'' Y + \frac{N_c}{3} \end{array} \right) \left(\begin{array}{c} R_{B'} \quad R_{\phi'} \parallel R_s \gamma'_s \\ I_{B'} Y_{B'} \quad I_{\phi'} Y_{\phi'} \parallel I_s Y_s \end{array} \right) \quad (3.1) \\
 &\times \sum_{K, \tilde{K}, \tilde{K}'} [K] ([\tilde{K}][\tilde{K}'])^{1/2} \left\{ \begin{array}{c} L \quad I \quad \tilde{K} \\ S \quad S_B \quad S_\phi \\ J_s \quad I'' \quad K \end{array} \right\} \left\{ \begin{array}{c} L' \quad I' \quad \tilde{K}' \\ S' \quad S_{B'} \quad S_{\phi'} \\ J_s \quad I'' \quad K \end{array} \right\} \tau_{K\tilde{K}\tilde{K}'LL'}^{\{II'Y\}},
 \end{aligned}$$

where the double-barred quantities are SU(3) isoscalar factors [15], quantities $[X]$ indicate representation multiplicities (for example, for angular momenta $[J] = 2J+1$), and the braced quantities are standard SU(2) $9j$ symbols.

In comparison, the original 2-flavor result [4] reads

$$\begin{aligned}
 S_{LL'SS'I_s J_s} &= \sum_{K, \tilde{K}, \tilde{K}'} [K] ([S_B][S_{B'}][S][S'][\tilde{K}][\tilde{K}'])^{1/2} \\
 &\times \left\{ \begin{array}{c} L \quad I_\phi \quad \tilde{K} \\ S \quad S_B \quad S_\phi \\ J_s \quad I_s \quad K \end{array} \right\} \left\{ \begin{array}{c} L' \quad I_{\phi'} \quad \tilde{K}' \\ S' \quad S_{B'} \quad S_{\phi'} \\ J_s \quad I_s \quad K \end{array} \right\} \tau_{K\tilde{K}\tilde{K}'LL'}. \quad (3.2)
 \end{aligned}$$

where [16] $\tau_{K\tilde{K}\tilde{K}'LL'} \equiv (-1)^{I_B - I_{B'} + I_\phi - I_{\phi'}} \tau_{K\tilde{K}\tilde{K}'LL'}^{\{I_\phi I_{\phi'} Y_\phi\}}$. Also shown in ref. [16] is the manner in which the flavor SU(3) factors reduce to isospin SU(2) factors in the large N_c limit.

Such factors are nontrivial because the $SU(3)$ representations and isoscalar factors are N_c dependent; for example, the nucleon resides not in a literal $\mathbf{8} = (1,1)$ in the usual weight notation, but rather “ $\mathbf{8}$ ” = $[1, (N_c - 1)/2]$.

One may also opt to express these amplitude expressions using t -channel quantities. Now the process is no longer expressed in the quantum numbers relevant to the s -channel process eq. (2.1), but those of the corresponding t -channel process

$$\phi + \bar{\phi}' \rightarrow \bar{B} + B' . \quad (3.3)$$

In particular, the quantum numbers S, S', I_s, J_s are traded for $J_\phi, J'_\phi, I_t, J_t$. Of course, the kinematic region for the literal on-shell process of eq. (3.3) [which requires momentum transfers obtained from the very large value $t \geq (m_{\bar{B}} + m_{B'})^2 = O(N_c^2)$] is very different from the one of eq. (2.1) [which only requires $O(N_c^0)$ momentum transfers]. Moreover, standard N_c power counting [21] shows that, while meson-baryon scattering amplitudes are $O(N_c^0)$, those for $\bar{B}B$ production are suppressed as e^{-N_c} . Nevertheless, the usual field-theoretic assumptions hold that the same amplitude (via analytic continuation of momenta) appears in both regions. We are interested only in the behavior of on-shell s -channel processes expressed in terms of t -channel quantum numbers.

This problem was originally addressed in ref. [5]; here we present corrected versions [10]¹ of the expressions derived in that work. In the 3-flavor case, one finds

$$\begin{aligned} S_{LL'J_\phi J_{\phi'} J_t R_t \gamma_t \gamma'_t I_t Y_t} &= (-1)^{S_{\phi'} - S_\phi + J_\phi - J_t} ([R_B][R_{B'}][J_\phi][J_{\phi'}])^{1/2} / [R_t] \\ &\times \sum_{\substack{I \in R_\phi, I' \in R_{\phi'}, \\ Y \in R_\phi \cap R_{\phi'}}} \left(\begin{array}{c} R_\phi \quad R_{\phi'}^* \\ IY \quad I', -Y \end{array} \left\| \begin{array}{c} R_t \quad \gamma_t \\ J_t \quad 0 \end{array} \right. \right) \left(\begin{array}{c} R_\phi \quad R_{\phi'}^* \\ I_\phi Y_\phi \quad I_{\phi'}, -Y_{\phi'} \end{array} \left\| \begin{array}{c} R_t \quad \gamma_t \\ I_t \quad Y_t \end{array} \right. \right) \\ &\times \left(\begin{array}{c} R_B^* \quad R_{B'} \\ S_B, -\frac{N_c}{3} \quad S_{B'}, +\frac{N_c}{3} \end{array} \left\| \begin{array}{c} R_t \quad \gamma'_t \\ J_t \quad 0 \end{array} \right. \right) \left(\begin{array}{c} R_B^* \quad R_{B'} \\ I_B, -Y_B \quad I_{B'} Y_{B'} \end{array} \left\| \begin{array}{c} R_t \quad \gamma'_t \\ I_t \quad Y_t \end{array} \right. \right) \\ &\times \sum_{K, \tilde{K}, \tilde{K}'} (-1)^{K + \tilde{K}' - \tilde{K} - \frac{Y}{2}} [K] ([\tilde{K}][\tilde{K}'])^{1/2} \left\{ \begin{array}{c} J_\phi \quad I \quad K \\ I' \quad J_{\phi'} \quad J_t \end{array} \right\} \left\{ \begin{array}{c} J_\phi \quad I \quad K \\ \tilde{K} \quad S_\phi \quad L \end{array} \right\} \left\{ \begin{array}{c} J_{\phi'} \quad I' \quad K \\ \tilde{K}' \quad S_{\phi'} \quad L' \end{array} \right\} \\ &\times \tau_{K \tilde{K} \tilde{K}' LL'}^{\{II'Y\}} , \end{aligned} \quad (3.4)$$

which reduces in the 2-flavor case, thanks to the 3-flavor $I_t = J_t$ and $Y_t = 0$ rules [10], to

$$\begin{aligned} S_{LL'J_\phi J_{\phi'} I_t J_t} &= \delta_{I_t J_t} (-1)^{I_{\phi'} + S_{\phi'} - S_\phi + J_\phi - J_t} ([S_B][S_{B'}][J_\phi][J_{\phi'}])^{1/2} / [I_t] \\ &\times \sum_{K, \tilde{K}, \tilde{K}'} (-1)^{K + \tilde{K}' - \tilde{K}} [K] ([\tilde{K}][\tilde{K}'])^{1/2} \left\{ \begin{array}{c} J_\phi \quad I_\phi \quad K \\ I_{\phi'} \quad J_{\phi'} \quad J_t \end{array} \right\} \left\{ \begin{array}{c} J_\phi \quad I_\phi \quad K \\ \tilde{K} \quad S_\phi \quad L \end{array} \right\} \left\{ \begin{array}{c} J_{\phi'} \quad I_{\phi'} \quad K \\ \tilde{K}' \quad S_{\phi'} \quad L' \end{array} \right\} \\ &\times \tau_{K \tilde{K} \tilde{K}' LL'} . \end{aligned} \quad (3.5)$$

Built into each of these expressions is the solitonic baryon wave function, which strictly speaking is adequate only in the large N_c limit. Each expression carries the correct $O(N_c^0)$

¹The phase in ref. [10] accidentally omitted the factor $(-1)^{\tilde{K}' - \tilde{K}}$ included here.

scaling as long as the reduced amplitudes τ are also $O(N_c^0)$. When $1/N_c$ corrections are included, the corrections are of two types: $O(1/N_c)$ corrections to the amplitudes τ [which are multiplicative in nature and do not change the group-theoretical structures exhibited in eqs. (3.1)–(3.5); for example, corrections to the profile functions of soliton models are of this type] and $1/N_c$ corrections with group-theoretical structures different from those in eqs. (3.1)–(3.5). Since $I_t = J_t$ (and $Y_t = 0$) is a direct consequence of these calculations, it follows that amplitudes with $I_t \neq J_t$ are necessarily subleading in $1/N_c$ [11].

We emphasize that solitonic wave functions, although part of the original derivations, are not essential to the process. They are invoked here merely to describe the historical path by which such relations first appeared. In the general large N_c approach, the only essential feature of the reduced amplitudes τ 's is that they are $O(N_c^0)$.

We close this section with a note on how the $I_t = J_t$ rule arises when one derives eq. (3.5) directly, rather than through a reduction of the 3-flavor result. There, the result holds almost trivially: The solitonic baryon wave functions of good quantum numbers $I_B = S_B$ are obtained [2–5] by rotating the canonical soliton through an $SU(2)$ element A using the rotation matrix $D_{I_{Bz}, -S_{Bz}}^{(I_B=S_B)}(A)$ of rank $I_B = S_B$. In the t channel one has such a rotation matrix for \bar{B} and one for B' . When these are combined using the standard identity [19]

$$\begin{aligned}
 & D_{-I_{Bz}, S_{Bz}}^{(I_{\bar{B}}=S_{\bar{B}})}(A) D_{I_{B'z}, -S_{B'z}}^{(I_{B'}=S_{B'})}(A) \\
 &= \sum_{\mathcal{J}} \left(\begin{array}{cc|c} I_{\bar{B}} & I_{B'} & \mathcal{J} \\ -I_{Bz} & I_{B'z} & -I_{Bz} + I_{B'z} \end{array} \right) \left(\begin{array}{cc|c} S_{\bar{B}} & S_{B'} & \mathcal{J} \\ S_{Bz} & -S_{B'z} & S_{Bz} - S_{B'z} \end{array} \right) D_{-I_{Bz}+I_{B'z}, S_{Bz}-S_{B'z}}^{(\mathcal{J})}(A), \quad (3.6)
 \end{aligned}$$

the same value of \mathcal{J} occurs in both the isospin and spin CGC. In light of the definitions of \mathbf{I}_t and \mathbf{J}_t in eq. (2.2), this expression shows that $\mathcal{J} = I_t = J_t$, a result that follows directly from the spin-flavor symmetry of the soliton.

4. Crossing relations

Two equivalent approaches lead to the amplitudes of eq. (3.4) or (3.5): One may work directly with the process written in terms of t -channel states, as in eq. (3.3), or derive a generic expression for crossing from the s to the t channel. The latter approach, pioneered in ref. [22], was also employed in ref. [5]. Since this is the means by which one may express the subleading in $1/N_c$ amplitudes in terms of s -channel quantities, we take some care to explain its derivation relevant to the present case.

4.1 2-flavor case

As seen in the previous section, the spin-only angular momenta \mathbf{S}, \mathbf{S}' are useful in the s channel, while the meson-only angular momenta $\mathbf{J}_\phi, \mathbf{J}_{\phi'}$ are useful in the t channel. Indeed, transforming between one order of coupling and another is precisely the original purpose

of $6j$ symbols [19]:

$$|j_1, (j_2 j_3) J_{23}, JM\rangle = \sum_{J_{12}} |(j_1 j_2) J_{12}, j_3, JM\rangle (-1)^{j_1+j_2+j_3+J} \sqrt{[J_{12}][J_{23}]} \left\{ \begin{matrix} j_1 & J_{12} & j_2 \\ j_3 & J_{23} & J \end{matrix} \right\}. \quad (4.1)$$

In our case, $j_1 \rightarrow S_B$, $j_2 \rightarrow S_\phi$, $j_3 \rightarrow L$, $J_{12} \rightarrow S$, $J_{23} \rightarrow J_\phi$, and $J \rightarrow J_s$, with analogous assignments for the primed quantities. Additional phases $(-1)^{L+S-J_s}$, $(-1)^{L+S_\phi-J_\phi}$, and $(-1)^{J_\phi+S_B-J_s}$ arise from noting that the orders of coupling $\mathbf{S} + \mathbf{L} \rightarrow \mathbf{J}_s$, $\mathbf{S}_\phi + \mathbf{L} \rightarrow \mathbf{J}_\phi$, and $\mathbf{S}_B + \mathbf{J}_\phi \rightarrow \mathbf{J}_s$, respectively, as given in eq. (4.1) are opposite those given in eq. (2.2). Using the symmetry properties of $6j$ symbols (invariance under exchanging two columns or the upper and lower entries of any two rows), one finds

$$S_{LL'J_\phi J_{\phi'} I_s J_s} = \sum_{S, S'} \sqrt{[S][S'][J_\phi][J_{\phi'}]} \times (-1)^{L-L'+S-S'} \left\{ \begin{matrix} J_s & J_\phi & S_B \\ S_\phi & S & L \end{matrix} \right\} \left\{ \begin{matrix} J_s & J_{\phi'} & S_{B'} \\ S_{\phi'} & S' & L' \end{matrix} \right\} S_{LL'SS'I_s J_s}. \quad (4.2)$$

The next step is to cross the t -channel quantities to an s -channel description [22]. The crossing at the computational level, in light of eq. (3.3), consists first of establishing a phase convention for exchanging bras with kets, and second of moving the CGC associated with $\bar{\phi}'$ and \bar{B} — the two that form I_t and the two that form J_t according to the last two definitions of eq. (2.2) — to the s -channel side of the equation. The phase convention for two flavors is

$$|II_z\rangle \leftrightarrow (-1)^{I+I_z} \langle I - I_z|, \quad (4.3)$$

and for three flavors we choose

$$|R, II_z, Y\rangle \leftrightarrow (-1)^{I_z + \frac{Y}{2} - \frac{1}{3}(2p+q)} \langle R^*, I - I_z, -Y|, \quad (4.4)$$

where the $SU(3)$ representation R in weight notation is (p, q) . The $SU(2)$ CGC are moved from one side of an equation to the other by means of their orthogonality relations, which leads to four CGC for isospin and four for spin, summed over all magnetic quantum numbers. But such invariants are again $6j$ symbols; specifically, one finds

$$S_{LL'J_\phi J_{\phi'} I_t J_t} = \sum_{I_s, J_s} [I_s][J_s] (-1)^{I_s+I_t+S_B-I_{\phi'}} (-1)^{J_s+J_t+S_B-J_{\phi'}} \times \left\{ \begin{matrix} S_{B'} & S_B & I_t \\ I_\phi & I_{\phi'} & I_s \end{matrix} \right\} \left\{ \begin{matrix} S_{B'} & S_B & J_t \\ J_\phi & J_{\phi'} & J_s \end{matrix} \right\} S_{LL'J_\phi J_{\phi'} I_s J_s}. \quad (4.5)$$

Combining eqs. (4.2) and (4.5) then gives

$$S_{LL'J_\phi J_{\phi'} I_t J_t} = \sum_{S, S', I_s, J_s} [I_s][J_s] ([J_\phi][J_{\phi'}][S][S'])^{1/2} (-1)^{I_t+I_s+J_t+J_s+L-L'+S-S'+2S_B-J_{\phi'}-I_{\phi'}} \times \left\{ \begin{matrix} S_{B'} & S_B & I_t \\ I_\phi & I_{\phi'} & I_s \end{matrix} \right\} \left\{ \begin{matrix} S_{B'} & S_B & J_t \\ J_\phi & J_{\phi'} & J_s \end{matrix} \right\} \left\{ \begin{matrix} J_s & J_\phi & S_B \\ S_\phi & S & L \end{matrix} \right\} \left\{ \begin{matrix} J_s & J_{\phi'} & S_{B'} \\ S_{\phi'} & S' & L' \end{matrix} \right\} S_{LL'SS'I_s J_s}. \quad (4.6)$$

Apart from a corrected phase, this expression was first derived in ref. [5]. Moreover, using the orthogonality properties of $6j$ symbols [19], eq. (4.6) may be inverted to give an inverse expression with precisely the same $6j$ symbols and phases:

$$\begin{aligned}
 S_{LL'SS'I_sJ_s} = & \sum_{J_\phi, J_{\phi'}, I_t, J_t} [I_t][J_t]([J_\phi][J_{\phi'}][S][S'])^{1/2} (-1)^{I_t+I_s+J_t+J_s+L-L'+S-S'+2S_B-J_{\phi'}-I_{\phi'}} \\
 & \times \left\{ \begin{matrix} S_{B'} & S_B & I_t \\ I_\phi & I_{\phi'} & I_s \end{matrix} \right\} \left\{ \begin{matrix} S_{B'} & S_B & J_t \\ J_\phi & J_{\phi'} & J_s \end{matrix} \right\} \left\{ \begin{matrix} J_s & J_\phi & S_B \\ S_\phi & S & L \end{matrix} \right\} \left\{ \begin{matrix} J_s & J_{\phi'} & S_{B'} \\ S_{\phi'} & S' & L' \end{matrix} \right\} S_{LL'J_\phi J_{\phi'} I_t J_t}.
 \end{aligned} \tag{4.7}$$

4.2 3-flavor case

Unfortunately, the 3-flavor crossing expressions for a process with external particles of fixed I and Y is not so straightforward to express in a simple closed form similar to eq. (4.7). The orthogonality relations for SU(2) CGC used to prove eq. (4.5) sum over the SU(2) magnetic quantum numbers I_z and J_z but not the Casimirs I and J . However, in the case of flavor SU(3) the quantum numbers summed in the analogous CGC orthogonality relations [20] also include values of I and Y , while in a given physical process (e.g., KN rather than $\pi\Lambda$), these values are specified by particular external states and are not summed. The 3-flavor crossing conditions must be written as a set of linear equations, with SU(3) isoscalar factors [those of eqs. (3.1) and (3.4)] on the two sides.

This result means that one cannot present an explicit expression for the crossing of a particular amplitude for which the SU(3) quantum numbers $R_s\gamma_s$ are specified. Fortunately, physical data specify quantum numbers such as I_s , J_s , and L , but not whether the scattering proceeds through an octet channel, for example. In fact, the 2-flavor crossing relations remain useful, for one may eliminate the SU(3) behavior simply by summing over all possible intermediate SU(3) quantum numbers, weighted by the appropriate SU(3) isoscalar factors.² Note that these are not just trivial projections from strange to non-strange amplitudes, but rather weighted averages of strangeness-containing amplitudes for which the SU(3) quantum numbers are irrelevant. Let us define in this way pure SU(2) amplitudes $\bar{S}_{LL'SS'I_sJ_s}$ and $\bar{S}_{LL'J_\phi J_{\phi'} I_t J_t}$:

$$\bar{S}_{LL'SS'I_sJ_s} \equiv \sum_{R_s, \gamma_s, \gamma'_s} \left(\begin{matrix} R_B & R_\phi & \parallel R_s \gamma_s \\ I_B Y_B & I_\phi Y_\phi & \parallel I_s Y_s \end{matrix} \right) \left(\begin{matrix} R_{B'} & R_{\phi'} & \parallel R_s \gamma'_s \\ I_{B'} Y_{B'} & I_{\phi'} Y_{\phi'} & \parallel I_s Y_s \end{matrix} \right) S_{LL'SS'J_s R_s \gamma_s \gamma'_s I_s Y_s}, \tag{4.8}$$

$$\bar{S}_{LL'J_\phi J_{\phi'} I_t J_t} \equiv \sum_{R_t, \gamma_t, \gamma'_t} \left(\begin{matrix} R_\phi & R_{\phi'}^* & \parallel R_t \gamma_t \\ I_\phi Y_\phi & I_{\phi'}^* Y_{\phi'} & \parallel I_t Y_t \end{matrix} \right) \left(\begin{matrix} R_{B'}^* & R_{B'} & \parallel R_t \gamma'_t \\ I_{B'}^* Y_{B'} & I_{B'} Y_{B'} & \parallel I_t Y_t \end{matrix} \right) S_{LL'J_\phi J_{\phi'} J_t R_t \gamma_t \gamma'_t I_t Y_t}. \tag{4.9}$$

Then the relation between the isospin amplitudes \bar{S} is the same as for the original SU(2) amplitudes S in eq. (4.6), except with the SU(2) crossing phases in eq. (4.3) replaced by

²A similar complication arose in the original crossing study of ref. [22], albeit at a lower level of complexity than considered here.

the ones for SU(3) given in eq. (4.4):

$$\begin{aligned}
 \bar{S}_{LL'J_\phi J_{\phi'} I_t J_t} &= \sum_{S, S', I_s, J_s} [I_s][J_s]([J_\phi][J_{\phi'}][S][S'])^{1/2} \\
 &\times (-1)^{I_t + I_s + J_t + J_s + L - L' + S - S' + 2S_B - J_{\phi'} + Y_{\phi'}/2} \\
 &\times \left\{ \begin{matrix} S_{B'} & S_B & I_t \\ I_\phi & I_{\phi'} & I_s \end{matrix} \right\} \left\{ \begin{matrix} S_{B'} & S_B & J_t \\ J_\phi & J_{\phi'} & J_s \end{matrix} \right\} \left\{ \begin{matrix} J_s & J_\phi & S_B \\ S_\phi & S & L \end{matrix} \right\} \left\{ \begin{matrix} J_s & J_{\phi'} & S_{B'} \\ S_{\phi'} & S' & L' \end{matrix} \right\} \bar{S}_{LL'SS'I_s J_s} .
 \end{aligned} \tag{4.10}$$

We see that a direct inversion of eqs. (4.8) and (4.9) is not possible, because the isoscalar factor orthogonality relations required to do so sum over externally fixed quantum numbers such as I_B, Y_B . However, as noted previously the full 3-flavor amplitudes themselves depend (implicitly) on these quantum numbers; only if one assumes the amplitudes are the same for all states in the SU(3) multiplets may one perform such an inversion and express the 3-flavor crossing relation in closed form. If one is unwilling to embrace this level of SU(3) symmetry but insists on retaining all SU(3) quantum numbers, the best one can do for a given process is obtain linear relations between the amplitudes expressed in the s and t channels. Fortunately, as discussed above, this degree of specificity is unnecessary for comparison with data; in section 6 we see that only eq. (4.10) is required to study, for example, KN scattering.

5. The $I_t = J_t$ rule and its corrections

Of course, in nature N_c is only 3, and a robust phenomenological analysis is not possible unless the structure of $1/N_c$ corrections is understood. As we have seen, the scattering amplitude expressions based upon chiral soliton models are quite impressive, but nevertheless hold only in the large N_c limit. To move beyond this point one requires additional input, which is provided by the operator approach. Starting with the ansatz (common to both soliton and quark models) that ground-state band baryons are completely symmetric under the combined spin-flavor symmetry, one divides the baryon wave function into N_c quark interpolating fields, each of which carries spin, flavor, and color fundamental representation indices. The color index, completely antisymmetrized among the N_c quarks, becomes irrelevant, and fundamental interactions with each quark may be categorized in terms of the *one-body* operators classified by spin-flavor:

$$\begin{aligned}
 J^i &\equiv \sum_\alpha q_\alpha^\dagger \left(\frac{\sigma^i}{2} \otimes \mathbb{1} \right) q_\alpha, \\
 T^a &\equiv \sum_\alpha q_\alpha^\dagger \left(\mathbb{1} \otimes \frac{\lambda^a}{2} \right) q_\alpha, \\
 G^{ia} &\equiv \sum_\alpha q_\alpha^\dagger \left(\frac{\sigma^i}{2} \otimes \frac{\lambda^a}{2} \right) q_\alpha,
 \end{aligned} \tag{5.1}$$

where the index α sums over the N_c quarks, σ^i are Pauli spin matrices, and λ^a are Gell-Mann flavor matrices. Each distinct operator may be written as a monomial in J, T , and

G of total order n (with $0 \leq n \leq N_c$) and is termed an n -body operator. A large subset of operators constructed in this way are redundant or give vanishing matrix elements due to group-theoretical constraints; the *operator reduction rules* derived in ref. [9] show how to remove systematically all such operators acting upon the ground-state band. Since each interaction requires a factor of $\alpha_s = O(1/N_c)$, operators composed of multiple one-body operators tend to be suppressed in powers of $1/N_c$. However, for the low-lying states in the ground-state band (N , Δ , etc.), I^a ($\equiv T^a$ for $a = 1, 2, 3$), J^i , and G^{i8} have matrix elements of $O(N_c^0)$ while G^{ia} with $a = 1, 2, 3$ and $a = 4, 5, 6, 7$ give matrix elements of $O(N_c^1)$ and $O(N_c^{1/2})$, respectively, T^a with $a = 4, 5, 6, 7$ gives $O(N_c^{1/2})$, and T^8 gives a part $O(N_c^1)$ times the identity operator (hence redundant) plus a part at $O(N_c^0)$ proportional to strangeness.

The original KSM theorem [8] (which was originally applied to nucleon-nucleon scattering) shows that amplitudes with $|I_t - J_t| = n$ scale at most as $O(1/N_c^n)$. The original KSM proof writes t -channel exchanges in terms of the one-body operators, and uses the fact that the only 2-flavor operator with $O(N_c^1)$ matrix elements is G^{ia} . If the indices on a string of G 's are summed, the operator reduction rules always turn out to generate a composite operator with subleading N_c counting; therefore, the leading operators are ones for which the spin and isospin indices on the G 's (of which there are equal numbers) are unsummed and symmetrized. A collection of \mathcal{J} G 's combined in this way thus gives a tensor with $I_t = J_t = \mathcal{J}$. Each contraction or one-body operator I^a or J^i instead of a G^{ia} costs a relative factor N_c , and therefore operators with $|I_t - J_t| = n$ are suppressed by at least a relative factor of $1/N_c^n$.

This proof was generalized [10] to three flavors by using (as noted above) that the isosinglet strangeness-conserving components G^{i8} and T^8 are effectively $O(1/N_c)$ compared to G^{ia} with $a = 1, 2, 3$ and hence do not spoil the theorem, while the strangeness-changing operators T^a , G^{ia} with $a = 4, 5, 6, 7$ provide a minimum $O(1/N_c^{1/2})$ suppression for each unit of strangeness change of the baryon, which is just Y_t . $1/N_c$ corrections to both the $I_t = J_t$ and $Y_t = 0$ rules are therefore straightforward to describe, using the operator formalism.

6. $1/N_c$ Corrections in the s channel

Section 5 shows how to incorporate $1/N_c$ corrections to meson-baryon scattering amplitudes, via t -channel exchanges with $I_t \neq J_t$ or $Y_t \neq 0$. Section 4 shows how to cross amplitudes written in terms of t -channel quantities into ones written in terms of s -channel quantities. Apart from managing the exceptionally cumbersome notation, nothing remains but to merge the two ideas. The t -channel amplitudes $S_{LL'J_\phi J_{\phi'} J_t R_t \gamma_t \gamma'_t I_t Y_t}$ (or $S_{LL'J_\phi J_{\phi'} I_t J_t}$ for the 2-flavor case) are suppressed by $N_c^{-|I_t - J_t|}$ (for non strangeness-exchanging processes) or $N_c^{-Y_t/2}$ (for strangeness-exchanging processes) compared to the leading-order $I_t = J_t$, $Y_t = 0$ amplitudes. Each t -channel amplitude may be inserted directly into eq. (4.10) plus eqs. (4.8) and (4.9) for the 3-flavor case [or just eq. (4.6) for the 2-flavor case] to give the corresponding s -channel suppressed amplitudes.

This is not to say that one cannot consider $1/N_c$ suppressions of orders higher than $1/N_c^{|I_t-J_t|_{\max}}$ or $1/N_c^{Y_t/2}$ for a given process. Each amplitude S carries a leading suppression of $1/N_c^\Delta$ with some definite Δ as determined by the $I_t=J_t$ and $Y_t=0$ rules; however, each one may also have subleading contributions $O(1/N_c^{\Delta+1})$ that are not discerned by this simple-minded analysis. These results may be summarized as just

$$\begin{aligned}
 S_{LL'J_\phi J_{\phi'} J_t R_t \gamma_t \gamma'_t I_t Y_t} &= O\left(1/N_c^{|I_t-J_t|}\right) \quad (Y_t=0), \\
 &= O\left(1/N_c^{Y_t/2}\right) \quad (Y_t \neq 0), \\
 &\rightarrow \sum_{\text{all except } L, L'} S_{LL'SS'J_s R_s \gamma_s \gamma'_s I_s Y_s}, \quad (6.1)
 \end{aligned}$$

with quantum numbers $S, S', J_s, R_s, \gamma_s, \gamma'_s, I_s, Y_s$ for the amplitudes on the right-hand side limited to those allowed by the group-theoretical constraints of eqs. (4.8) and (4.10).

In the case of scattering with spinless pions ($S_\phi=S_{\phi'}=0, I_\phi=I_{\phi'}=1$), eq. (4.7) reduces to the forms used to study the phenomenology of $\pi N \rightarrow \pi N, \pi \Delta$ scattering processes in refs. [11, 13]. The amplitude $S_{LL'S_B S_{B'} I_s J_s}$ receives a correction

$$-\frac{1}{N_c^{|I_t-J_t|}} s_{I_t LL'}^{t(J_t-I_t)} = (-1)^{L+L'} (9[L][L'] [S_B]^2 [S_{B'}]^2)^{-1/4} [I_t][J_t] S_{LL'LL'I_t J_t}. \quad (6.2)$$

Finally, we present one explicit example of the formalism that has not previously been considered in the literature: $1/N_c$ corrections to the process $\pi N \rightarrow \rho N$. As seen in ref. [17], the processes $\pi N \rightarrow \pi \pi N$ are dominated for large N_c by resonant $\pi N \rightarrow \pi \Delta, \rho N$, or ωN intermediate states, and moreover, branching fractions for such processes have been extracted from raw scattering data. However, ref. [17] worked only with the leading [$O(N_c^0)$] amplitudes and found the results in many cases (for predicting branching ratios of given baryon resonances to particular final states) to be rather inconclusive. In addition to the large uncertainties in the data, a principal culprit for this imprecision lay in the omission of $1/N_c$ corrections. The $1/N_c$ -suppressed amplitudes are clearly significant, because they were shown in several cases to be of the right order of magnitude to explain discrepancies between data and the leading-order predictions. A full reanalysis of the sort performed in ref. [17] but including $1/N_c$ corrections is of course far outside of our current scope, but we can at least show how quickly the onerous expressions obtained above simplify for a physical case.

Consider two simple cases, both with $I_s = J_s = \frac{1}{2}$ and the initial πN in a state of relative $L=0$. Then the final ρN can either be in a state of relative $L'=0$ when $S' = \frac{1}{2}$ (the S_{11} partial wave), or $L'=2$ when $S' = \frac{3}{2}$ (the SD_{11} partial wave). Then eq. (4.7) gives

$$\begin{aligned}
 S_{11}^{(\pi N)(\rho N)_1} &= +\sqrt{\frac{3}{2}} S_{000111} + \frac{1}{2} S_{000101}, \\
 SD_{11}^{(\pi N)(\rho N)_3} &= -\sqrt{\frac{3}{2}} S_{020111} - \frac{1}{2} S_{020101}, \quad (6.3)
 \end{aligned}$$

where we use the notation of ref. [17]: The superscript is $2S'$. As a reminder, the last two indices of amplitudes S on the right-hand side are $I_t J_t$, so the second amplitude in each

case is $1/N_c$ suppressed. More amplitudes arise for higher spins and higher partial waves, but like this example, the explicit expressions tend to be quite simple in general.

As discussed above, if one is not concerned with the specific $SU(3)$ quantum numbers $R_s \gamma_s$, one may apply eq. (4.10) directly. As an example, consider KN scattering; for spinless mesons, $S = S_B$ and $S' = S'_B$, and the s -channel amplitude $\bar{S}_{LL'SS'I_s J_s}$ is denoted in the literature by $(LL')_{I_s, 2J_s}$. When specialized to the S -wave case ($L = L' = 0$), eq. (4.10) simply gives

$$\bar{S}_{000000} = \frac{1}{\sqrt{2}} (S_{01} + 3S_{11}) , \tag{6.4}$$

$$\bar{S}_{000010} = \frac{1}{\sqrt{2}} (S_{01} - S_{11}) . \tag{6.5}$$

Recalling that the last two indices of the t -channel amplitudes on the left-hand side are I_t and J_t , we note that eq. (6.4) is $O(N_c^0)$ and eq. (6.5) is $O(1/N_c)$. From the second of these it follows that $S_{01} = S_{11}$ up to $O(1/N_c)$ corrections. In fact, available partial-wave data³ supports this approximate equality: Both the real and imaginary parts of S_{01} and S_{11} have the same signs and basic shapes as functions of s , and are approximately equal for large s , with differences in the 1.5-2.0 GeV region that can be attributed to relative $O(1/N_c)$ corrections.

7. Conclusions

In this paper we have examined in detail the procedure for crossing between s - and t -channel quantum numbers for meson-baryon scattering in the context of the $1/N_c$ expansion. The s -channel quantum numbers are the ones used most frequently for describing physical scattering processes. However, the t -channel quantum numbers are the ones most convenient for quantifying $1/N_c$ power suppressions, using the degree of violation of the $I_t = J_t$ or $Y_t = 0$ rules.

We have given explicit expressions for crossing any given amplitude in the t channel in terms of a linear combination of amplitudes in the s channel and vice-versa, in the case of two quark flavors. In the 3-flavor case, unless one assumes $SU(3)$ symmetry for all meson-baryon amplitudes, one obtains not a single closed-form crossing solution, but a series of linear equations that impose constraints on the amplitudes.

Finally, we have shown how the complicated expressions obtained here simplify to those used previously, and exhibited as an explicit example a simple novel case, $\pi N \rightarrow \rho N$ formed in the S_{11} channel. Such expressions as obtained here may be used in a detailed phenomenological analysis, including subleading $1/N_c$ corrections, for processes such as $\pi N \rightarrow \text{multi-}\pi N$ or KN scattering.

³The SAID Partial-Wave Analysis website (<http://gwdac.phys.gwu.edu/>), Center for Nuclear Studies, George Washington University.

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